Int. J. Heat Mass Transfer. Vol. 34, No. 4/5, pp. 1003–1008, 1991
Printed in Great Britain

0017-9310/91 \$3.00+0.00
Pergamon Press plc

Radiation heat exchange between non-diffuse gray surfaces separated by isothermal absorbing–emitting gas

Z. ZAVARGO, M. DJURIC and M. NOVAKOVIC
Faculty of Technology, University of Novi Sad, Novi Sad, Yugoslavia

(*Received for publication* 22 *June* 1990)

Abstract—In many industrial thermal systems and plants thermal radiation is one of the major means of energy transfer. Analysis of radiation heat exchange in enclosures is often simplified introducing the following assumptions: emissivity and reflectivity are independent of direction and wavelength; the reflected energy is diffuse and uniform. These approximations greatly simplify the enclosure theory but as it can be seen (Rohsenow and Hartnett, *Handbook of Heat Transfer*. McGraw-Hill, New York (1973)) many engineering materials show direction-dependent properties. The non-diffuse nature of thermal radiation is often approximated by a model based on the assumption that whole emission is diffuse while the reflection is partly specular and partly diffuse (Sarofim and Hottel, *J. Heat Transfer* **88**(1), 37–44 (1966)). However, many materials have also non-diffuse emission (Rohsenow and Hartnett, *Handbook of Heat Transfer*. McGraw-Hill, New York (1973)). In this work a model based on non-diffuse emission and reflection of surfaces separated by isothermal absorbing–emitting gas is presented. The radiation from surfaces and gas volume is simulated by an artificial, random process, while the determination of beam extinguishing place enables the calculation of geometrical system characteristics (like absorption factor) as well as the formulation of heat balance equations. The model presented is tested on the simple geometry of a cube and the results show the influence of non-diffuse emission and reflection as well as the presence of an intervening medium in such systems.

INTRODUCTION

RADIATION between non-diffuse gray surfaces is often approximated by a model proposed by Sarofim and Hottel [1]. This model is based on diffuse emission and non-diffuse reflection. Non-diffuse reflection is approximated by the following assumption:

$$\rho = 1-\varepsilon = \rho_s+\rho_d$$

where ρ_s and ρ_d are the specular and diffuse components of reflection ρ.

However, in proper analysis non-diffuse emission must be also taken in account [2–4].

In this work heat exchange between surfaces with non-diffuse reflection separated by isothermal absorbing–emitting gas was investigated. The model was tested on a geometry of a cube for three materials: black surfaces, CuO and Cr. In the case of CuO the coefficient of emission ε decreases with β (angle between normal on surface and direction of emission). In the case of Cr the coefficient of emission ε increases with β [5].

For the numerical calculation of radiation heat exchange the statistical Monte Carlo method was adopted [6–8].

RADIATION HEAT EXCHANGE

One could write the following equation of heat exchange between surfaces A_i and A_j:

$$Q_{i \rightleftarrows j} = A_i B_{ij} \bar{\varepsilon}_i (\sigma T_i^4 - \sigma T_j^4) \quad (1)$$

where B_{ij} is the absorption factor and A_i and A_j can be defined as the fraction of radiation leaving surfaces A_i that reaches surface A_j and is absorbed after all possible reflections. This equation has the same form as that for the case of transparent gas. The difference is in B_{ij}. $\bar{\varepsilon}_i$ represents the mean coefficient of emission over $\beta = 0-\pi/2$. The heat exchange equation between surface A_i and gas volume V is

$$Q_{i \rightleftarrows g} = A_i B_{ig} \bar{\varepsilon}_i (\sigma T_i^4 - \sigma T_g^4). \quad (2)$$

The equation of heat exchange between the gas volume and surface A_i is

$$Q_{g \rightleftarrows i} = 4aVB_{gi}(\sigma T_g^4 - \sigma T_i^4) \quad (3)$$

where B_{gi} is the absorption factor gas–surface and a the gas absorption coefficient. In the case of an isotherm gas the following equation is valid [9]:

$$Q_{i \rightleftarrows g} = Q_{g \rightleftarrows i}. \quad (4)$$

The conservation principle is valid so it can be written in the case of a single gas zone as

$$B_{ij} + B_{ig} = 1 \quad (5)$$

$$B_{gi} + B_{gg} = 1. \quad (6)$$

MODEL

The considered model is based on the following assumptions:

NOMENCLATURE

A	area
B	absorption factor
E	energy
i	radiation intensity, index
L	length of beam
N_a, N_t	number of rays absorbed, total number of rays
Q	heat
R	random numbers
T	temperature
V	volume.

Greek symbols

α	angle between emitted and reflected surfaces
β	angle between normal on surfaces and direction of emission
ε, $\bar{\varepsilon}$	coefficient of emission, mean coefficient of emission
θ, θ'	azimuth in emitted and reflected plane
ρ, ρ_s, ρ_d	coefficient of reflection, specular, diffuse component of reflection
ρ'_s, ρ'_d	fraction of specular and diffuse component of reflection.

(1) All surfaces are gray.

(2) Emission is nondiffuse, $\varepsilon = \varepsilon(\beta)$.

(3) Reflection is nondiffuse. Part of these is reflected specularly and the other part diffusely

$$\rho(\beta) = 1-\varepsilon(\beta) = \rho(\beta)\rho'_s+\rho(\beta)\rho'_d.$$

(4) The medium separated the surfaces of the isothermal absorbing–emitting gas.

In the above expression ρ'_s and ρ'_d represent the fraction of the specular and diffuse components of reflection.

THE MONTE CARLO METHOD

The Monte Carlo method is a statistical method by which the history of each individual 'energy bundle' (ray) is followed from the point of emission to the point of absorption or escape from the system.

In an enclosure consisting of n surfaces the emitted flux from surface A_i is

$$E_i\,dA_i = \left[\int_i \varepsilon(\beta) i \sin\beta\cos\beta\,d\beta\,d\theta\right] dA_i. \quad (7)$$

According to the definition of absorption

$$B = \frac{E'_{ij}\,dA_i}{\int_0^{\pi/2}\int_0^{2\pi}\varepsilon(\beta) i \sin\beta\cos\beta\,d\beta\,d\theta} \quad (8)$$

where E'_{ij} represents the fraction of energy leaving A_i that reaches A_j and is absorbed. Dividing the emitted flux into n individual rays, $e_i = E_i\,dA_i/N_i$, which represents 'individual energy'. In the case where the number of rays are finite

$$B_{ij} = N_a(j)/N_t \quad (9)$$

where N_t is the total number of random rays from i and $N_a(j)$ the total number of rays out of these which are absorbed on surface A_j.

PROCEDURE FOR FINDING ABSORPTION FACTOR

Emission from surface

The coordinate of the random emitted ray was found applying two random numbers R_x and R_y and the direction of the ray by another two random numbers R_β and R_θ

$$\theta = 2\pi R_\theta \quad (10)$$

$$\int_0^{\pi/2}\varepsilon(\beta)\sin\beta\cos\beta\,d\beta]R_\beta = \int_0^{\beta}\varepsilon(\beta)\sin\beta\cos\beta\,d\beta. \quad (11)$$

Once the random ray is defined one must find the strike on the plane r (see Fig. 1). Having in mind Bouger's exponential law of absorption the random length is

$$L_R = -\frac{1}{a}\ln R_l. \quad (12)$$

Comparing L_T (total distance between coordinate of

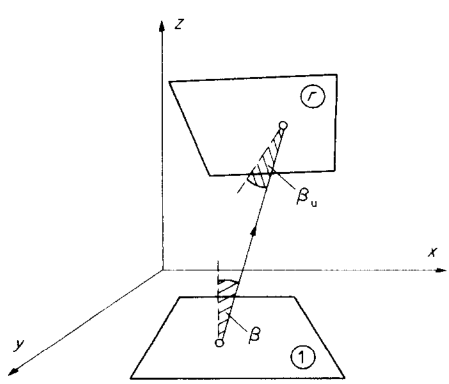

FIG. 1. The random ray strike on plane r.

random ray and point of strike) to L_R the further fate of the ray can be determined:

for $L_T > L_R$ ray reaches the surface r

for $L_T > L_R$ ray is absorbed in gas.

In the case of $L_T < L_R$ the number of strike surface–gas is increased by one

$$SG = SG + 1.$$

A new random ray is chosen. In the case of $L_T > L_R$ the coefficient of emission can be found, $\varepsilon = \varepsilon(\beta_u)$ (β_u is the angle between the normal on plane r and the emitted ray). The further fate of the beam is decided regarding the following conditions:

$0 < R_s \leqslant \varepsilon(\beta_u)$ ray is absorbed on plane r

$\varepsilon(\beta_u) < R_s \leqslant \varepsilon(\beta_u) + \rho_s$ ray is reflected specularly

$\varepsilon(\beta_u) + \rho_s < R_s \leqslant 1$ ray is reflected diffusely.

In the case of absorption the number of strike surface–surface is increased by one

$$SS(r) = SS(r) + 1.$$

In the case of specular reflection the beam is reflected in accordance with the laws which are valid for mirrorlike surfaces.

In the case of diffuse reflection we must choose a random azimuth angle θ' in plane r. The corresponding azimuth in the emitting plane is

$$\tan\theta = \cos\alpha \tan\theta' \qquad (13)$$

where α represents the angle between the emitted and the reflected plane.

Emission from gas

The coordinate of the random ray from the gas is chosen with three random numbers R_x, R_y and R_z. The direction of emission is determined by two random numbers R_β and R_θ [10]

$$\cos\beta = 1 - 2R_\beta \qquad (14)$$

$$\theta = 2\pi R_\theta. \qquad (15)$$

After determining the point of strike one must find L_T (distance between the origin of the random ray and the point of strike).

Comparing L_T and L_R the further fate of the beam can be determined in the same way as in the case of the emission from the plane.

In the case of absorption in the gas the number of strike which determines the absorption factor gas–gas is increased by one

$$GG = GG + 1.$$

Otherwise, the further fate of the beam is determined. In the case of absorption the number of strike gas–surface is increased by one

$$GS(r) = GS(r) + 1.$$

In the case of reflection the history of the beam is followed until it is absorbed whether in gas or on a surface.

RESULTS

The numerical experiment of radiation heat exchange between non-diffuse surfaces separated by isothermal absorbing–emitting gas was carried out on the geometry of a cube.

In the case of a cube with a single gas zone the following equations are valid:

$$B_{16} + 4B_{12} + B_{1g} = 1 \qquad (16)$$

$$6B_{g1} + B_{gg} = 1. \qquad (17)$$

The B_{16}, B_{12} are related to an opposite and adjacent view, respectively, B_{1g} surface–gas, B_{g1} gas–surface and B_{gg} gas–gas view. Further, for the case of absolute black surfaces the following equation can be written:

$$B_{gg} = 1 - \frac{6B_{1g}}{4ax}$$

where x presents the edge of a cube.

In Tables 1 and 2 the results are presented for the case of a black cube filled with gas of optical thickness $ax = 1$. These results were compared with results acquired on the base of mean beam length.

The results for the case when one surface of a cube is non-diffuse and the others are black are presented in Tables 3 and 4. In the case of a non-diffuse surface CuO, the following relation is valid:

$$B_{12} > B_{16}.$$

In the case of transparent gas it was opposite. It can be explained by the fact that for the CuO coefficient of emission ε decreases with β. However, in the pres-

Table 1. Emission from surface

B	MC	AN	Δ%
B_{12}	0.122	0.120	1.67
B_{16}	0.068	0.068	0
B_{1g}	0.444	0.452	−1.77
B_{gg}	0.334	0.332	

MC, Monte Carlo results (this procedure).
AN, analytical (based on mean beam length).

Table 2. Emission from gas

B	MC	AN	Δ%
B_{g1}	0.111	0.113	−1.77
B_{gg}	0.334	0.322	3.72
B_{1g}	0.444		

MC, Monte Carlo results (this procedure).
AN, analytical (based on mean beam length).

Table 3. Emission surface is CuO, others are black

B	0.4	0.6	1	4	6	8
			ax			
B_{12}	0.1677	0.1521	0.1195	0.0466	0.0325	0.0247
B_{16}	0.1114	0.0878	0.0717	0.003	0.0003	0.0
B_{1g}	0.2178	0.3037	0.4503	0.8105	0.8697	0.9012

Table 4. One surface is CuO, others are black, $\rho'_d = 0.6$

B	0.4	0.6	1	4	6	8
			ax			
B_{g1}	0.0096	0.0081	0.0063	0.00355	0.00185	0.0017
B_{g2}	0.1621	0.1474	0.1254	0.05262	0.03738	0.0277
B_{g6}	0.1559	0.14325	0.1136	0.05345	0.03495	0.0261
B_{gg}	0.1861	0.25905	0.3785	0.73252	0.81368	0.86112

Table 5. Emission surface is Cr, others are black

B	0.4	0.6	1	4	6	8
			ax			
B_{12}	0.1584	0.1437	0.1195	0.0466	0.0325	0.0247
B_{16}	0.1389	0.1091	0.0717	0.0029	0.0003	0.0
B_{1g}	0.2275	0.3161	0.4503	0.8105	0.8697	0.9012

Table 6. One surface is Cr, others are black, $\rho'_d = 0.6$

B	0.4	0.6	1	4	6	8
			ax			
B_{g1}	0.1017	0.0939	0.08235	0.0387	0.028	0.0208
B_{g2}	0.1464	0.13525	0.1152	0.05167	0.03655	0.02848
B_{g6}	0.1472	0.13295	0.11305	0.047	0.03285	0.0275
B_{gg}	0.1655	0.2321	0.3438	0.07076	0.79295	0.83778

ence of intervening gas a ray passes a greater distance going to the opposite surface than to the adjacent one. Due to this fact more radiation is absorbed in the gas.

Although the emission from gas is not favoured in any direction, due to reflection from surface 1, B_{g2} is greater than B_{g6} (shorter distance, 1–2). Due to reflection the absorption factor B_{g1} has the lowest value.

Tables 7 and 8 present the results for the case when all surfaces of a cube were nondiffuse (CuO and Cr).

CONCLUSION

In this work the radiation between surfaces with direction-dependent emission and diffuse+specular reflection was investigated. The medium between surfaces was isothermal absorbing–emitting gas.

The model was tested on the geometry of a cube. Two non-diffuse materials were chosen: one with increasing value of ε with β(CuO) and the other with decreasing value of ε with β(Cr).

Table 7. All surfaces: CuO, $\rho'_d = 0.6$

B	$ax = 1$	Transparent gas
	Emission from surface	
B_{11}	0.015	0.045
B_{12}	0.0963	0.183
B_{16}	0.0698	0.221
B_{1g}	0.530	—
	Emission from gas	
B_{g1}	0.0974	—
B_{gg}	0.4156	—

Table 8. All surfaces: Cr, $\rho'_d = 0.6$

B	$ax = 1$	Transparent gas
	Emission from surface	
B_{11}	0.0083	0.158
B_{12}	0.0162	0.168
B_{16}	0.0107	0.168
B_{1g}	0.9162	·
	Emission from gas	
B_{g1}	0.01538	—
B_{gg}	0.9077	—

With greater optical thickness the nature of non-diffuse emission becomes irrelevant.

REFERENCES

1. A. F. Sarofim and H. C. Hottel, Radiative exchange among non-Lambert surfaces, *J. Heat Transfer* **88**(1), 37–44 (1966).
2. Z. Zavargo and M. Djuric, Mathematical model of radiation heat exchange between gray direction dependent surfaces, *Proc. 5th Yug.-Austr.-Ital. CEC*, Portoroz, September (1986).
3. J. T. Bevans and D. K. Edwards, Radiation exchange in enclosure with directional wall properties, ASME Paper No. 64 WA/HT-52 (1964).
4. R. P. Bobco, A script-F matrix formulation for enclosure with arbitrary surface emission and reflection characteristics, *J. Heat Transfer* **93**, 33–40 (1971).
5. W. Rohsenow and J. P. Hartnett, *Handbook of Heat Transfer*. McGraw-Hill, New York (1973).
6. J. Doenecke, Application of the Monte Carlo method for radiation exchange in spacecraft, *J. Spacecraft* **10**(10), 652–657 (1973).
7. J. Toor and R. Viskanta, A numerical experiment of radiation heat exchange by the Monte Carlo method, *Int. J. Heat Mass Transfer* **11**, 883–897 (1968).
8. J. Toor and R. Viskanta, Effect of direction dependent properties on radiation interchange, *J. Spacecraft* **5**(6), 742–743 (1969).
9. D. A. Lihov, Review of furnace design methods, *Trans. J. Chem. Engng* **55**, 225–242 (1977).
10. M. Novakovic, M. Djuric and Z. Zavargo, Radiation heat transfer in a black enclosure with gray gas, *Termotehnika* No. 4, 271–282 (1986).

ECHANGE THERMIQUE RADIATIF ENTRE DES SURFACES GRISES NON DIFFUSANTES SEPAREES PAR UN GAZ ISOTHERME ABSORBANT ET EMISSIF

Résumé—Dans beaucoup de systèmes thermiques industriels le rayonnement est le mode principal de transfert de chaleur. L'analyse de l'échange radiatif dans des cavités est souvent simplifiée en introduisant les hypothèses suivantes: l'émissivité et la réflectivité sont indépendantes de la direction et de la longueur d'onde; l'énergie réfléchie est diffuse et uniforme. Ces approximations simplifient beaucoup car comme on peut le voir (Rohsenow et Hartnett, *Handbook of Heat Transfer*. McGraw-Hill, New York (1973)), de nombreux matériaux ont des propriétés dépendant de la direction. La nature non diffuse du rayonnement thermique est souvent approchée par un modèle basé sur l'hypothèse d'une émission diffuse tandis que la réflexion est partiellement spéculaire et partiellement diffuse (Sarofim et Hottel, *J. Heat Transfer* **88**(1), 37–44 (1966)). Néanmoins beaucoup de matériaux ont aussi une émission non diffuse. Le présent modèle est présenté avec une émission et une réflexion non diffuse de surfaces séparées par un gaz isotherme absorbant et émissif. Le rayonnement des surfaces et du volume du gaz est simulé par un mécanisme artificiel et aléatoire, tandis que la détermination de l'extinction du rayonnement rend possible le calcul des caractéristiques du système (comme le facteur d'absorption) ainsi que la formulation des équations du bilan énergétique. Le modèle présenté est testé sur la géométrie simple d'un cube et les résultats montrent l'influence de l'émission et de la réflexion non diffuse aussi bien que du gaz intervenant.

STRAHLUNGSWÄRMEAUSTAUSCH ZWISCHEN NICHT-DIFFUS STRAHLENDEN, GRAUEN, DURCH EIN ISOTHERMES ABSORBIEREND/EMITTIERENDES GAS GETRENNTEN OBERFLÄCHEN

Zusammenfassung—Bei vielen industriellen thermischen Prozessen und Anlagen ist die Wärmestrahlung von wesentlicher Bedeutung für den Energietransport. Der Strahlungswärmeaustausch in Hohlräumen wird oft unter den folgenden vereinfachenden Annahmen untersucht: Emissions- und Reflexionsvermögen sind richtungs- und wellenlängenunabhängig; die Energie wird diffus und gleichförmig reflektiert. Diese Näherungen vereinfachen in starkem Maße die Hohlraumtheorie. Viele Konstruktionsmaterialien zeigen jedoch richtungsabhängige Eigenschaften (Rohsenow und Hartnett, *Handbook of Heat Transfer*. McGraw-Hill, New York (1973)). Das nicht-diffuse Wesen der Wärmestrahlung wird oft mit Hilfe eines Modells angenähert, das darauf beruht, daß die gesamte Emission diffus, die Reflexion dagegen teilweise gerichtet und teilweise diffus ist (Sarofim und Hottel, *J. Heat Transfer* **88**(1), 37–44 (1966)). Viele Materialien zeigen jedoch auch eine nicht-diffuse Emission (siehe Rohsenow und Hartnett). In der vorliegenden Arbeit wird ein Modell vorgestellt, das auf nicht-diffuser Emission und Reflexion an Oberflächen beruht, welche von einem isothermen absorbierend/emittierenden Gas getrennt sind. Die Strahlung von den Oberflächen und aus dem Gasvolumen wird mit Hilfe eines künstlichen Zufallsprozesses simuliert. Die Bestimmung des Ortes der Strahlabsorption ermöglicht die Berechnung geometrischer Systemgrößen (wie zum Beispiel des Absorptionsvermögens) sowie die Formulierung von Wärmebilanzgleichungen. Dieses Modell wird an der einfachen Geometrie eines Würfels erprobt. Die Ergebnisse zeigen den Einfluß der nicht-diffusen Emission und Reflexion wie auch den Einfluß der Gasfüllung in solchen Systemen.

РАДИАЦИОННЫЙ ТЕПЛООБМЕН МЕЖДУ НЕРАССЕИВАЮЩИМИ СЕРЫМИ ПОВЕРХНОСТЯМИ, РАЗДЕЛЕННЫМИ ИЗОТЕРМИЧЕСКИМ ПОГЛОЩАЮЩИМ-ИСПУСКАЮЩИМ ГАЗОМ

Аннотация—Во многих промышленных тепловых системах и установках тепловое излучение является одним из основных способов переноса энергии. Анализ радиационного теплопереноса в полостях часто упрощается посредством следующим предположений: излучательная и отражательная способности не зависят от направления и длины волны; излучение отражается диффузно. Указанные приближения намного упростили теорию теплообмена в полостях, однако, как показано (Rohsenow and Hartnett, *Handbook of Heat Transfer*. McGraw-Hill, New York (1973)), свойства многих технических материалов зависят от направления. Недиффузный характер отражения теплового излучения часто приближенно выражается моделью, основанной на предположении, что все испускание является диффузным, в то время как отражение—частично зеркальным и частично диффузным (Sarofim and Hottel, *J. Heat Transfer* **88**(1), 37–44 (1966)). В то же время существует много материалов с недиффузным испусканием. В данном исследовании описывается модель недиффузного испускания и отражения от поверхностей, разделенных изотермическим поглощающим-испускающим газом. Излучение от поверхностей и объема газа моделируется случайным процессом, а определение места затухания потока излучения позволяет рассчитать характеристики геометрической системы (такие как коэффициент поглощения) и вывести уравнения теплового баланса. Адекватность предложенной модели проваряется на простой геометрии куба; полученные результаты показывают влияние недиффузного излучения и отражения, а также наличия промежуточной среды в рассматриваемых системах.